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Targeting spatiotemporal patterns in extended systems with multiple coexisting attractors

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We set up adaptive control algorithms which can be used to achieve control to desired attractors in spatially extended systems. Traditional adaptive control methods often fail in such systems due to the presence of multiple coexisting attractors that lead to a high probability of the system getting trapped in an undesired attractor despite the application of control. We use quenching techniques to achieve control in such difficult scenarios. When the control parameter evolves through parameter regions that lead to undesired attractors, the control parameter is changed sufficiently fast so that the system does not get time to get trapped in these attractors, but gets quenched instead to the desirable attractor. The rate of change of the parameter is guided by using variable stiffness of control. We demonstrate the efficacy of our technique in a system of coupled sine-circle maps. Further, such variable stiffness schemes can also be used to step up the efficiency of adaptive control algorithms by making frequent suitable changes in the stiffness of control during the control dynamics. This strategy is very successful in reducing the time required to achieve control, while maintaining the stability of the control dynamics.

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Considerable recent research effort has focused on mechanisms of control in strongly nonlinear systems which typically display a diversity of dynamical behavior in parameter space. Such methods aim to reach and maintain a fixed dynamical activity (the ‘‘target’’) in systems intrinsically capable of very complicated behavior [1–8]. In addition to attempts directed towards controlling low-dimensional nonlinear systems [1–4], substantial efforts have gone into the control of spatiotemporal behavior in extended systems [5–8]. These range from the stabilization of periodic patterns in optical turbulence [5] and the selection of spatiotemporal current densities in semiconductors [6] to the control of buckling beam systems using smart matter [7] and the targeting of spatiotemporal patterns in coupled map lattices [8].

The control problem is particularly difficult in extended systems that possess a multiplicity of coexisting attractors. The reason for this is that to obtain the target, which is one of these coexisting attractors, the control dynamics not only need to evolve to the desired parameter values, via methods such as adaptive control [1], but it must evolve in such a way that the state of the system either remains in the basin of attraction of the targeted state, or evolves to the appropriate basin of attraction. We indicate below a potent method for achieving control in such difficult control situations. In this method the rate of change of parameter in different regions of parameter space is guided by varying the ‘‘stiffness’’ of control, such that the control parameter is evolved very fast through parameter regions, which might settle down to undesired attractors so that the system is rapidly ‘‘quenched’’ to the desired attractor.

First, let us recall the adaptive control algorithm, proposed in Ref. [1] and developed and extended in Refs. [2–5] and [8]. The procedure utilizes an error signal proportional to the difference between the goal output and the actual output of the system. The error signal drives the evolution of the parameters which readjust so as to reduce the error to zero. Specifically, in a general N-dimensional nonlinear dynamical system described by the evolution equation $\dot{\mathbf{X}}=\mathbf{F}(\mathbf{X};\mu;t)$, where $\mathbf{X}\equiv(X_1,X_2,\ldots,X_N)$ are the state variables and μ is the parameter whose value determines the nature of the dynamics, the adaptive control applies a feedback loop in order to drive the system parameter (or parameters) to the value(s) required, so as to achieve a desired target state via the equation

$$\dot{\mu}=\gamma(\mathcal{P}^{\star}-\mathcal{P}), \qquad (1)$$

where $\mathcal{P}^{\star}$ is the target value of some variable or property $\mathcal{P}$ (which could be a function of several variables) and the value of γ indicates the *stiffness of control*. Here the *error signal* $\mathcal{P}^{\star}-\mathcal{P}$ drives the system to the target state. The control stiffness γ *regulates the strength of feedback* and thus, *determines how rapidly the system is controlled.* When the system achieves the target the control equation ‘‘switches off’’ (as the error signal becomes zero).

VARIABLE STIFFNESS ALGORITHMS TO ACHIEVE QUENCHING

In situations where a system has to traverse large parameter regions where it can get trapped in undesirable attractor basins enroute to the target, traditional adaptive control methods as stated above will fail, but variable stiffness can still make control achievable. The basic idea is to guide the system very quickly through treacherous terrain (by increasing the stiffness of control) so that it is ‘‘quenched’’ to the basin of attraction of the target state. Once inside the control basin, i.e., the set of initial points from which fixed stiffness control is achievable, the stiffness is lowered so that the sys-

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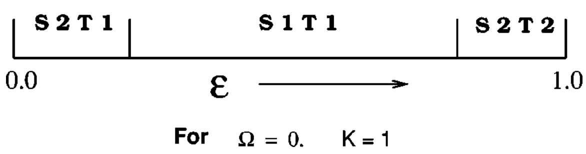

FIG. 1. Schematic diagram (to scale) showing the phases for SCML, with respect to coupling parameter ϵ (for $\Omega=0$, $K=1$), obtained from spatial period two initial conditions, $\dots x_1 x_2 x_1 x_2 \dots$, with $x_1+x_2=1.0$.

tem does not oscillate wildly about the target and leave the control basin again.

We demonstrate this control principle in a lattice of coupled sine-circle maps targeting different spatiotemporal behaviors [8]. This system is capable of exhibiting a rich variety of spatiotemporal patterns [9], including coexisting basins of attraction [10], and thus provides a good testing ground for the technique. Note that the method is quite general and can be directly applied to other extended systems as well.

The time evolution of a coupled sine-circle map lattice (SCML) is given by

$$x_{n+1}(i)=(1-\epsilon)f[\Omega,K,x_n(i)]+\frac{\epsilon}{2}\{f[Kx_n(i-1)] + f[\Omega,K,x_n(i+1)]\} \quad \text{Mod } 1, \tag{2}$$

where n is the discrete time index and i is the site index ($i=1,\dots,N$, where N is the lattice size). The local map is

$$f(x,\Omega,K)=x+\Omega-\frac{K}{2\pi}\sin(2\pi x), \tag{3}$$

where $0\leqslant x\leqslant 1$. K indicates the strength of the nonlinearity and ϵ gives the strength of coupling among neighbors. The system supports various dynamical phases, such as the synchronized fixed point, i.e., spatial period 1 temporal period 1 (S1T1), spatial period 2 temporal period 1 (S2T1), and spatial period 2 temporal period 2 (S2T2). Figure 1 schematically shows a slice of ϵ parameter space (with $\Omega=0,K=1$), demarcating the regions of stability of the S2T2, S2T1, and S1T1 solutions obtained from period 2 initial conditions (see Ref. [9] for a detailed phase diagram). Importantly, note that these S2T1 and S2T2 regions coexist with the S1T1 solution, which in fact has a very large basin of attraction. This makes conventional adaptive control methods unfeasible for targeting the S2T1 and S2T2 states in this system, as the control basin for these states is very small. Since the control difficulties encountered here are representative of the generic problems arising due to coexisting attractors in extended systems that display hysteresis, we will use this situation as a test-bed for the quenched adaptive feedback method [11].

To target spatiotemporal patterns we must use spatial or temporal feedback $\mathcal{P}^\star-\mathcal{P}$, specifically tailored for the distinctive characteristics of the desired targeted pattern. In addition, the feedback should be simply defined, without the explicit knowledge of the system's equations of motion, in order to have greater utility in experimental applications. Further, it must not be measurement intensive, i.e., it must not entail monitoring a large number of sites. In fact, here we will only use information from a *single* (arbitrary) site for the necessary feedback.

As a representative example, we demonstrate the control procedure on the coupling parameter ϵ, which implies the following:

$$\epsilon_{n+1}=\epsilon_n-\gamma\times\text{sgn}(\epsilon_{mid}-\epsilon_n)\times(\Delta x-\Delta x^*), \tag{4}$$

with Δx^* being the target value of the local expansionlike quantity Δx. The factor $\text{sgn}(\epsilon_{mid}-\epsilon_n)$ takes care of the sign of the control feedback, with ϵ_{mid} being a very rough estimate of the mid-point of the parameter region supporting the targeted state. In order to target S2T1. $\Delta x=|x_{n+1}(i_c+1)-x_n(i_c)|$ and for control to a S2T2 state, $\Delta x=|x_{n+1}(i_c)-x_n(i_c)|$, where i_c is a *single* (arbitrary) lattice site monitored for feedback. These error signals distinguish clearly between the targets and are not satisfied by any of the spatiotemporal behavior, other than the targeted one.

Now this is a difficult control situation, as the multiplicity of coexisting attractors here implies that reaching the right parameter is not enough to ensure control. For instance, in the parameter region supporting the S2T1 and S2T2 states, the fixed point is also a stable state with a very large basin of attraction. In fact, any generic random initial condition will go to a synchronized fixed point. Only period 2 initial lattices will be attracted to the S2T1 or S2T2 states. Thus, conventional control fails in such cases. For example, if the S2T2 phase is targeted from the S2T1 region of parameter space or vice versa (with the initial state in the basin of attraction of the spatial period two state), control cannot be achieved due to the large intervening fixed point regime (see Fig. 1) in which the state is unable to escape synchronization. The usual method of using noise to jolt the system out of undesired trapping basins enroute to the target does not work here, as these basins are quite extensive in parameter space and very stable. Thus the only way to achieve the desired target is to *quench* the system so that the system does not have time to respond to the changed parameter by settling down to the undesired synchronized fixed point. This quenching is achieved using large stiffness of control.

In our method we start the control procedure with very large initial stiffness and then use the following algorithm to maintain an acceptable level of stiffness: (i) Estimate the controlled parameter ϵ with initial stiffness γ_0 (γ_0 large); (ii) Test: if the estimated ϵ is not in the range $\epsilon_{low}<\epsilon<\epsilon_{high}$, reduce stiffness [$\gamma$ in Eq. (4)] by a predetermined factor (for instance, reduce to half); (iii) Repeat until $\epsilon_{low}<\epsilon<\epsilon_{high}$. The only inputs in this algorithm are the limiting bounds for the controlled parameter, ϵ_{high} and ϵ_{low}. These can be easily set to be the limiting values of the parameter, e.g., in this case ϵ_{low} is naturally 0 and ϵ_{high} is 1.

Now this variable stiffness algorithm can effectively take the system from the S2T2 state to the control basin of the targeted S2T1 state by adjusting the controlled parameter ϵ so fast that the system does not have a chance to synchronize

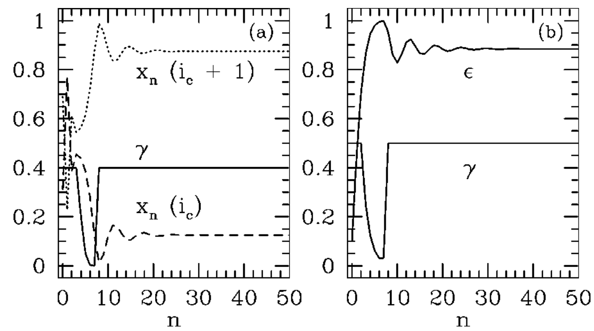

FIG. 2. (a) Plots of the evolution of $x_n(i_c)$ and $x_n(i_c+1)$ (dashed lines), where i_c is the monitored site, as well as γ (bold line), for control to the S2T1 state of a SCML via the quenching algorithm. Here, initial $\epsilon_0=0.9$ (i.e. in the S2T2 regime), $\gamma_0=0.4$, $\epsilon_{mid}\sim 0$, and $\Delta x^*=0.75$. (b) Plots of the evolution of ϵ and γ, for control to the S2T2 state of a SCML via the quenching algorithm. Here, initial $\epsilon_0=0.1$ (i.e. in the S2T1 regime), $\gamma_0=0.5$, $\epsilon_{mid}\sim 1$, and $\Delta x^*=0.75$. The initial lattice ($N=100$) has spatial period two in both cases.

and get trapped in the fixed point region (see Fig. 2). The control is achieved in only ~ 20 steps.

In a similar fashion, the method successfully takes a system from the S2T1 state to the S2T2 state, by rapidly dragging the system into the control basin of the targeted region. Complete control is again achieved in ~ 20 steps (see Fig. 3). Note that the control time is quite the same for lattices of different sizes.

We must however note that while run-time control does not necessitate computations based on dynamical equations, it is necessary at the outset to chart out the rough bifurcation diagram of the system. Indeed, one cannot gauge the *need* for quenching without some knowledge of the layout of the dynamical phases and their basins of attraction in parameter space. However, this knowledge need not be detailed and is

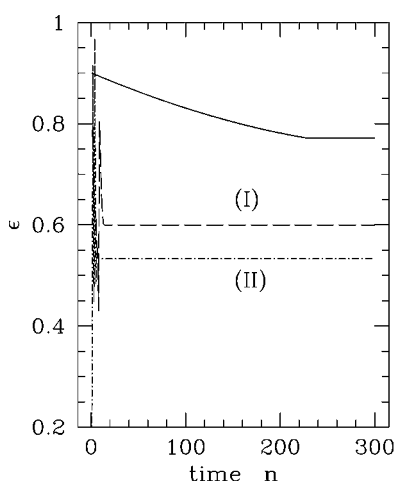

FIG. 3. Plots of the evolution of ϵ from initial $\epsilon_0=0.9$, for control to the S1T1 state of a SCML ($N=100$) via variable stiffness schemes I (long dash) and II (short dash), and via the fixed stiffness algorithm (solid line). Initial γ_0 is 0.001 in scheme I and 5 in scheme II.

not used during runtime. Moreover, in the case of real experiments, this knowledge is readily obtainable, even in the absence of a model.

VARIABLE STIFFNESS ALGORITHMS FOR ENHANCED EFFICIENCY

Now we will try to use the variable stiffness of control to achieve much more *efficient* control. The control (or recovery) time τ, defined as the time required to reach the desired state within finite precision, is crucially dependent on the value of stiffness γ. While for small γ the control time is inversely proportional to γ [3], beyond an optimal stiffness, in most systems, increasing γ actually *retards recovery* or *renders the control dynamics unstable* as the system swings wildly about the target without ever being ''damped'' onto the target [2,3]. So there is a *trade-off between stability and speed of control*. This crucial dependence of control times on the stiffness of control is the key behind our scheme to enhance the efficacy of the adaptive control algorithm by tuning stiffness γ to some optimal value at each point in the control path.

The principal idea is as follows: we would like to *optimize progress towards the goal by making frequent suitable changes in the stiffness of control*. The purpose is to achieve a predetermined accuracy in minimum time. This entails monitoring at each step how far we can safely increase the value of γ for the next step. Two distinct strategies can be employed to achieve this: (i) Start with very low control stiffness (which is guaranteed to yield stable control) and increase it to the maximum acceptable level; (ii) Start with very high control stiffness and then come down to an acceptable level. The implementation of both strategies involve a test which returns information on the error incurred in taking higher γ. It is achieved here via two schemes, which we again demonstrate on the SCML, targeting a spatiotemporal fixed point (S1T1).

Specifically, for instance, to reach and maintain the S1T1, one can employ the following control strategy: here the target is $x_{n+1}(i)-x_n(i)=0$ and $x_n(i+1)-x_n(i)=0$ for all sites i at all times n. We can choose the spatial property $\mathcal{P}=x_n(i+1)-x_n(i)$, for control to the synchronized state as it distinguishes between S1T1 and the neighboring S2T1 state [which, while having the property $x_{n+1}(i)-x_n(i)=0$, has $x_n(i+1)-x_n(i)\neq 0$]. The controlled parameter ϵ then evolves utilizing an error signal Δx, given by $\Delta x=x_n(i_c+1)-x_n(i_c)$, where i_c is the single site being monitored for feedback for adaptive control.

Significantly, this method can be implemented without explicit computation of the dynamics during run-time control, and just one site (and its local neighborhood) is monitored to obtain the required feedback, and this is capable of regulating the entire lattice. On this adaptive algorithm we can implement schemes I and II for varying stiffness γ in order to reduce control time without compromising stability.

Scheme I. In this method, at every point in the control dynamics we set control stiffness at some very low value and then increase it to the maximum acceptable level at that point. An estimate of the error incurred by taking higher stiffness is obtained by using a ''local lyapunov''-like exponent. If this is within a preassigned acceptable limit of accuracy we increase the stiffness of control for the next adaptive control step. The idea then is that one cannot estimate the acceptable level of stiffness *a priori*, and thus starts from a very low level which guarantees stable control and lets the algorithm find an acceptably high stiffness as the system evolves.

To implement this general strategy utilizing *no knowledge of the evolution equations*, i.e., using only the time series data of a particular variable, we do the following: (i) Initially choose a small stiffness $\gamma=\gamma_0$ with $\gamma_0\rightarrow 0$. Small γ *guarantees* stable successful control even if very slow [3]; (ii) If $|\gamma\times[x_n(i_c)-x_{n-1}(i_c)]|<\delta$ where δ is a predetermined accuracy, we double γ (note that $|x_n(i_c)-x_{n-1}(i_c)|$ is ''local lyapunov''-like factor and indicates the ''local chaos'' or ''local expansion properties'' at the current phase point in the control path [11]); (iii) Repeat step 2 till the accuracy requirement is violated.

Extensive numerics indicate that control times are improved dramatically by the method. For instance, starting with $\gamma_0=0.001$, control time with a fixed stiffness algorithm is ~ 225, while this variable stiffness algorithm yields control in times of the order of ten steps (see Fig. 3).

Scheme II. This scheme is very simply stated as follows: an estimate of the controlled parameter [via Eq. (1)] is made and if this estimate exceeds a preassigned upper or lower bound, the stiffness is reduced, or else it's kept at the original high value. Thus, we start with very high control stiffness and then come down to a level in keeping with the demands of stability and the operational range of parameter–phase space. Specifically then, we vary stiffness by the following algorithm: (i) Set control stiffness γ to some high value γ_0; (ii) Estimate the value of the subsequent adjustment in the controlled parameter ϵ, obtained via, $\epsilon'=\epsilon_n-\gamma[\Delta x_n(i_c)-\Delta x^*]$; (iii) If ϵ' is larger than ϵ_{high} or less than ϵ_{low}, then $\gamma\rightarrow\gamma/2$; (iv) Go to step 2 and repeat step 3 if necessary.

Extensive numerics clearly show the success of the above strategy. Though the stiffness adjustments are infrequent, recovery times are improved dramatically. For instance, control of S1T1 from a random initial lattice with $\epsilon=0.9$, now takes only ~ 11 iterations (see Fig. 3). Note that the control time is quite the same for lattices of different sizes.

Thus, both of these variable stiffness control algorithms have the desired effect of tuning the value of γ so that the controlled dynamics yields a spatiotemporal fixed point in times much shorter than that required for fixed stiffness algorithms.

In summary, we have suggested how variable stiffness adaptive control algorithms can be used to achieve control in situations where control fails with fixed control stiffness, such as in the presence of coexisting attractors, a phenomena widespread in extended systems. In such difficult control scenarios we use variable stiffness to guide the rate of change of the parameter and achieve control by changing the parameter sufficiently fast so that the system does not have time to get trapped in any undesired attractor. Further, we show how such variable stiffness schemes can be used to step up the efficiency of control by making frequent suitable changes in the stiffness of control, resulting in huge gains in efficiency vis-a-vis fixed control stiffness algorithms. Our methods are simple and can be implemented without detailed knowledge of the system. We therefore hope they will be of utility in practical contexts.

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